revisionvillage

Mathematics: Analysis & Approaches HL

1 Page Formula Sheet

Topic 1: Number & Algebra

1.2	
nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
Sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
1.3	
nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
Sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, r \neq 1$
1.8	
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}, \lvert r\rvert < 1$
1.4	
Compound Interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV is future value, PV is present value, n is number of years, k is compounding periods per year, r% is nominal annual interest rate
1.5	
Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b,\ a, b > 0, a \neq 1$
1.7	
Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
Exponential and logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$
1.9	
Binomial theorem, $n \in \mathbb{N}$	$(a+b)^n =$ $a^n + {}^n\mathrm{C}_1 a^{n-1}b + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$ ${}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$
1.10	
Combinations	${}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$
Permutations	${}^n\mathrm{P}_r = \frac{n!}{(n-r)!}$
Extension of the binomial theorem, $n \in \mathbb{Q}$	$(a+b)^n =$ $a^n\left(1 + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2 + \cdots\right)$
1.12	
Complex numbers	$z = a + bi$
1.13	
Modulus-argument (polar), exponential (Euler) form	$z = r(\cos\theta + \mathrm{i}\sin\theta) = re^{\mathrm{i}\theta} = r\mathrm{cis}\theta$
1.14	
De Moivre's theorem	$[r(\cos\theta + \mathrm{i}\sin\theta)]^n =$ $r^n(\cos n\theta + \mathrm{i}\sin n\theta) = r^n e^{\mathrm{i}n\theta} = r^n \mathrm{cis} n\theta$

Topic 2: Functions

2.1	
Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.6	
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
2.7	
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$
2.12	
Sum and product of the roots of polynomial equations	$\sum_{r=0}^{n} a_r x^r = 0 \Rightarrow$ sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Topic 3: Geometry & Trigonometry – Prior Learning

Area: parallelogram	$A = bh$, where b is base, h is height
Area: triangle	$A = \frac{1}{2}(bh)$, where b is base, h is height
Area: trapezoid	$A = \frac{1}{2}(a+b)h$, where a and b are the parallel sides, h is height
Area: circle	$A = \pi r^2$, where r is radius
Circumference: circle	$C = 2\pi r$, where r is radius
Volume: cuboid	$V = lwh$, l is length, w is width, h is height
Volume: cylinder	$V = \pi r^2 h$, where r is radius, h is height
Volume: prism	$V = Ah$, A is area of cross-section, h is height
Surface area: cylinder curve	$A = 2\pi rh$, where r is radius, h is height
Distance between 2 points (x_1, y_1); (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint of (x_1, y_1); (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Topic 3: Geometry & Trigonometry

3.1	
Distance between 2 points (x_1, y_1, z_1); (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of midpoint of (x_1, y_1, z_1); (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume: right-pyramid	$V = \frac{1}{3}Ah$, where A is area of base, h is height
Volume: right-cone	$V = \frac{1}{3}\pi r^2 h$, where r is radius, h is height
Surface area: cone curve	$A = \pi rl$, where r is radius, l is slant height
Volume: sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
Surface area: sphere	$A = 4\pi r^2$, where r is the radius
3.2	
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab\sin C$
3.4	
Length of an arc	$l = r\theta$ (where r is the radius, θ is the angle measured in radians)
Area of a sector	$A = \frac{1}{2}r^2\theta$ (where r is the radius, θ is the angle measured in radians)
3.5	
Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.6	
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$
3.9	
Reciprocal trig identities	$\sec\theta = \frac{1}{\cos\theta}$; $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$
Pythagorean identities	$1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \mathrm{cosec}^2\theta$
3.10	
Compound angle identities	$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
Double angle identity for tan	$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$
3.12	
Magnitude of vector	$\lvert\boldsymbol{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
3.13	
Scalar product	$\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$ $\boldsymbol{v}\cdot\boldsymbol{w} = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$ where θ is angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Angle between two vectors	$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert}$
3.14	
Vector line equation	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$
Parametric form of the equation of a line	$x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$
Cartesian equations of a line	$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$
3.16	
Vector product	$\boldsymbol{v}\times\boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$ $\lvert\boldsymbol{v}\times\boldsymbol{w}\rvert = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\sin\theta$ where θ is angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Area: parallelogram	$A = \lvert\boldsymbol{v}\times\boldsymbol{w}\rvert$, where $\boldsymbol{v}$ and $\boldsymbol{w}$ form 2 adjacent sides of parallelogram
3.17	
Vector equation of a plane	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$
Equation of a plane (normal vector)	$\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n}$ (using normal vector)
Cartesian equation of a plane	$ax + by + cz = d$

Topic 4: Statistics & Probability

4.2	
Interquartile range	$\mathrm{IQR} = Q_3 - Q_1$
4.3	
Mean, $\bar{x}$, of a data set	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ where $n = \sum_{i=1}^{k} f_i$
4.5	
Probability of an event A	$\mathrm{P}(A) = \frac{n(A)}{n(U)}$
Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
4.6	
Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
Conditional probability	$\mathrm{P}(A\vert B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$
4.7	
Expected value of a discrete random variable X	$\mathrm{E}(X) = \sum_{i=1}^{k} x_i\,\mathrm{P}(X = x_i)$
4.8	
Binomial distribution	$X \sim \mathrm{B}(n, p)$
Mean	$\mathrm{E}(X) = np$
Variance	$\mathrm{Var}(X) = np(1-p)$
4.12	
Standardized normal variable	$z = \frac{x - \mu}{\sigma}$
4.13	
Bayes' theorem	$\mathrm{P}(B\vert A) = \frac{\mathrm{P}(B)\mathrm{P}(A\vert B)}{\mathrm{P}(B)\mathrm{P}(A\vert B) + \mathrm{P}(B')\mathrm{P}(A\vert B')}$ $\mathrm{P}(B_i\vert A) = \frac{\mathrm{P}(B_i)\mathrm{P}(A\vert B_i)}{\mathrm{P}(B_1)\mathrm{P}(A\vert B_1) + \mathrm{P}(B_2)\mathrm{P}(A\vert B_2) + \mathrm{P}(B_3)\mathrm{P}(A\vert B_3)}$

Topic 4: Statistics & Probability

4.14	
Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$
Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i(x_i - \mu)^2}{n}}$
Linear transformation: single random variable	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$
Expected value of a continuous random variable X	$\mathrm{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)\mathrm{d}x$
Variance	$\mathrm{Var}(X) = \mathrm{E}[(X - \mu)^2] = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
Variance of a discrete random variable X	$\mathrm{Var}(X) = \sum(X - \mu)^2\,\mathrm{P}(X = x)$ $= \sum x^2\,\mathrm{P}(X = x) - \mu^2$
Variance of a continuous random variable X	$\mathrm{Var}(X) = \int_{-\infty}^{\infty}(X - \mu)^2 f(x)\mathrm{d}x$ $= \int_{-\infty}^{\infty} x^2 f(x)\mathrm{d}x - \mu^2$

Topic 5: Calculus

5.12	
Derivative of $f(x)$ from first principles	$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$
5.3	
Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.6	
Standard derivatives	$f(x) = \sin x \Rightarrow f'(x) = \cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = e^x \Rightarrow f'(x) = e^x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
Product rule	$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
5.15	
Standard derivatives	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x\tan x$ $f(x) = \mathrm{cosec}\,x \Rightarrow f'(x) = -\mathrm{cosec}\,x\cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\mathrm{cosec}^2 x$ $f(x) = a^x \Rightarrow f'(x) = a^x(\ln a)$ $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x\ln a}$ $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1 - x^2}}$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1 - x^2}}$ $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1 + x^2}$
5.9	
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$
Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2}\lvert v(t)\rvert\,\mathrm{d}t$
Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t)\,\mathrm{d}t$
5.5	
Integral of x^n	$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C, n \neq -1$
Area between a curve $f(x)$ and the x-axis, $f(x) > 0$	$A = \int_a^b y\,\mathrm{d}x$
5.10	
Standard Integrals	$\int \frac{1}{x}\mathrm{d}x = \ln\lvert x\rvert + C$ $\int \sin x\,\mathrm{d}x = -\cos x + C$ $\int \cos x\,\mathrm{d}x = \sin x + C$ $\int e^x\,\mathrm{d}x = e^x + C$
5.15	
Standard Integrals	$\int a^x\,\mathrm{d}x = \frac{1}{\ln a}a^x + C$ $\int \frac{1}{a^2 + x^2}\mathrm{d}x = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}}\mathrm{d}x = \arcsin\left(\frac{x}{a}\right) + C, \lvert x\rvert < a$
5.16	
Integration by parts	$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$
5.11	
Area of region enclosed by a curve and x-axis	$A = \int_a^b \lvert y\rvert\,\mathrm{d}x$
5.17	
Area of region enclosed by a curve and y-axis	$A = \int_a^b \lvert x\rvert\,\mathrm{d}y$
Volume of revolution about the x or y-axes	$V = \int_a^b \pi y^2\,\mathrm{d}x$ or $V = \int_a^b \pi x^2\,\mathrm{d}y$
5.18	
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$ where h is a constant (step length)
Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x)\mathrm{d}x}$
5.19	
Maclaurin series	$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \cdots$
Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \cdots$ $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$